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GS-324

VI Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - VIII

(CBCS) (Fresh+Repeaters) (2016-17 & Onwards)

Time : 3 Hours Max. Marks : 70

Instruction : Answer **all** questions.

PART - A

1. Answer **any five** questions. **5x2=10**

(a) Evaluate $\lim_{z\to 1+i} (z^2+2z)$.

(b) Show that $\left|\frac{z-2}{z+2}\right| = 3$ represents a circle.

(c) Show that $u = e^x \sin y + x^2 - y^2$ is harmonic.

(d) Define cross ratio of four points.

(e) State Liouville's Theorem.

(f) Evaluate $\oint (\bar{z})^2 dz$ around the circle $|z| = 1$

(g) Write Euler modified formula.

(h) State Runge- Kutta Method of order 4.

PART- B

Answer **four** full Questions. **4x10=40**

2. (a) Show that $\arg\left(\frac{z-1+i}{z+1}\right) = \frac{\pi}{4}$

(b) Prove that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$

OR

3. (a) Prove that $\lim_{z\to 0}\left(\frac{\bar{z}}{z}\right)$ does not exists.

(b) Show that $f(z)$ = sin z is analytic and hence prove that $f'(z) = \cos z$.

4. (a) Show that $u = y^3 - 3x^2y$ is harmonic and find its harmonic conjugate.

(b) If $f(z) = u + iv$ is an analytic function then prove that the curves $u(x,y) = c_1$, $v(x,y) = c_2$ form two orthogonal families.

OR

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5. (a) Find the analytic function $f(z)=u+iv$ given that $u-v=e^x(\cos y-\sin y)$.

(b) If $f(z)=u+iv$ is analytic then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}|f(z)|\right]^2=|f'(z)|^2$

6. (a) Evaluate $\int_{(0,3)}^{(2,4)}[(2y+x^2)dx+(3x-y)dy]$ using the substitution $x=2t,\ y=t^2+3$.

(b) State and Prove Cauchy's Integral Formula.

OR

7. (a) Evaluate $\oint_c \frac{z-4}{z(z^2+9)}dz$ where c is the circle $|z|=1$.

(b) State and prove Cauchy's Integral Theorem.

8. (a) Prove that the Bilinear Transformation preserves the cross ratio of four points.

(b) Discuss the transformation $w=z^2$.

OR

9. (a) Find the Bilinear Transformation which maps $z=1,e^1,-1$ on to $w=i,0,-i$.

(b) Show that the transformation $W=1/z$ transforms a circle to a circle or to a straight line.

PART- C

Answer **two** full questions. **2x10=20**

10. (a) Find the root of the equation $x^3-4x+1=0$ by Regula-falsi method upto three decimal places.

(b) Use Newton-Raphson Method to find a real root of the equation. $x^3-9x+1=0$ near $x=3$.

OR

11. (a) Solve by Guass- Jacobi method.

$10x+2y+z=9$

$x+10y-z=-22$

$-2x+3y+10z=22$

(b) Find the largest Eigen Value of the matrix.

$$A=\begin{pmatrix}1&6&1\\1&2&0\\0&0&2\end{pmatrix}$$

12. (a) Use Taylor's series method to find y at $x=0.1$ considering terms upto the third degree given $\frac{dy}{dx}=x^2+y^2$ and $y(0)=1$.

(b) Using Euler's modified method, find $y(0.2)$ given $\frac{dy}{dx}=x-y^2$, $y(0)=1$ taking $h=0.1$.

OR

13. (a) Use Euler's Method to solve $\frac{dy}{dx}=x+y$, $y(0)=1$ for $x=0.0\ (0.2)1.0$

(b) Using Runge-Kutta Method find $y(0.2)$ for $\frac{dy}{dx}=x+y$; $y(0)=1$ taking $h=0.2$.

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